PROBLEM 4-6a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and θ_4. Determine the Grashoff condition.

Given:

Link 1	$d := 6 \cdot in$	Link 2	$a := 2 \cdot in$	
Link 3	$b := 7 \cdot in$	Link 4	$c := 9 \cdot in$	$\theta_2 := 30 \cdot deg$

Solution: See figure below for one possible solution. Also see Mathcad file P0406a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Locate pivot O_4 on the x-axis at a distance from the origin equal to the given length of link 1.
5. Draw a circle with center at O_4 and a radius equal to the given length of link 4.
6. The two intersections of the circles (if any) are the two solutions to the position analysis problem, crossed and open. If the circles don't intersect, there is no solution.
7. Draw links 3 and 4 in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure their angles θ_3 and θ_4 with respect to the x-axis. From the solution below,

OPEN $\theta_{31} := 88.84 \cdot deg$

$\theta_{41} := 117.29 \cdot deg$

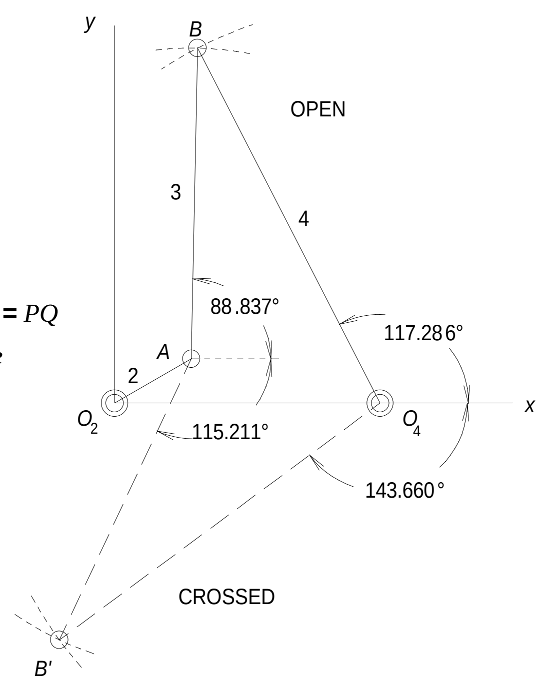
CROSSED $\theta_{32} := 360 \cdot deg - 115.21 \cdot deg$ $\theta_{32} = 244.790\ deg$

$\theta_{42} := 360 \cdot deg - 143.66 \cdot deg$ $\theta_{42} = 216.340\ deg$

8. Check the Grashof condition.

$$Condition(a,b,c,d) := \left| \begin{array}{l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"}\ \ if\ \ SL < PQ \\ return\ \text{"Special Grashof"}\ \ if\ \ SL = PQ \\ return\ \text{"non-Grashof"}\ \ otherwise \end{array} \right.$$

$$Condition(a,b,c,d) = \text{"Grashof"}$$



PROBLEM 4-7a

Statement: The link lengths and value of θ_2 for some fourbar linkages are defined in Table P4-1. The linkage configuration and terminology are shown in Figure P4-1. For row *a*, find all possible solutions (both open and crossed) for angles θ_3 and θ_4 using the vector loop method. Determine the Grashof condition.

Given: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 3 $b := 7 \cdot in$ Link 4 $c := 9 \cdot in$ $\theta_2 := 30 \cdot deg$

Two argument inverse tangent

$$atan2(x, y) := \begin{vmatrix} return\ 0.5 \cdot \pi \ \ if\ x = 0 \wedge y > 0 \\ return\ 1.5 \cdot \pi \ \ if\ x = 0 \wedge y < 0 \\ return\ atan\left(\left(\dfrac{y}{x}\right)\right) \ \ if\ x > 0 \\ atan\left(\left(\dfrac{y}{x}\right)\right) + \pi \ \ otherwise \end{vmatrix}$$

Solution: See Mathcad file P0407a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$K_1 := \frac{d}{a} \qquad K_2 := \frac{d}{c} \qquad K_3 := \frac{a^2 - b^2 + c^2 + d^2}{2 \cdot a \cdot c}$$

$$K_1 = 3.0000 \qquad K_2 = 0.6667 \qquad K_3 = 2.0000$$

$$A := cos(\theta_2) - K_1 - K_2 \cdot cos(\theta_2) + K_3 \qquad A = -0.7113$$

$$B := -2 \cdot sin(\theta_2) \qquad B = -1.0000$$

$$C := K_1 - (K_2 + 1) \cdot cos(\theta_2) + K_3 \qquad C = 3.5566$$

2. Use equation 4.10b to find values of θ_4 for the open and crossed circuits.

Open: $\theta_{41} := 2 \cdot \left(atan2\left(2 \cdot A, -B - \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\qquad \theta_{41} = 477.286\ deg$

$\theta_{41} := \theta_{41} - 360 \cdot deg$ $\qquad \theta_{41} = 117.286\ deg$

Crossed: $\theta_{42} := 2 \cdot \left(atan2\left(2 \cdot A, -B + \sqrt{B^2 - 4 \cdot A \cdot C}\right)\right)$ $\qquad \theta_{42} = 216.340\ deg$

3. Determine the values of the constants needed for finding θ_3 from equations 4.11b and 4.12.

$$K_4 := \frac{d}{b} \qquad K_5 := \frac{c^2 - d^2 - a^2 - b^2}{2 \cdot a \cdot b} \qquad K_4 = 0.8571$$

$$K_5 = -0.2857$$

$$D := cos(\theta_2) - K_1 + K_4 \cdot cos(\theta_2) + K_5 \qquad D = -1.6774$$

$$E := -2 \cdot sin(\theta_2) \qquad E = -1.0000$$

$$F := K_1 + (K_4 - 1) \cdot cos(\theta_2) + K_5 \qquad F = 2.5906$$

4. Use equation 4.13 to find values of θ_3 for the open and crossed circuits.

Open: $\theta_{31} := 2 \cdot \left(atan2\left(2 \cdot D, -E - \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\theta_{31} = 448.837\,deg$

$\theta_{31} := \theta_{31} - 360 \cdot deg$ $\theta_{31} = 88.837\,deg$

Crossed: $\theta_{32} := 2 \cdot \left(atan2\left(2 \cdot D, -E + \sqrt{E^2 - 4 \cdot D \cdot F} \right) \right)$ $\theta_{32} = 244.789\,deg$

5. Check the Grashof condition.

```
Condition(a, b, c, d) := | S ← min(a, b, c, d)
                         | L ← max(a, b, c, d)
                         | SL ← S + L
                         | PQ ← a + b + c + d − SL
                         | return "Grashof"  if SL < PQ
                         | return "Special Grashof"  if SL = PQ
                         | return "non-Grashof"  otherwise
```

Condition(a, b, c, d) = "Grashof"

PROBLEM 4-9a

Statement: The link lengths, value of θ_2, and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, draw the linkage to scale and graphically find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given: Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

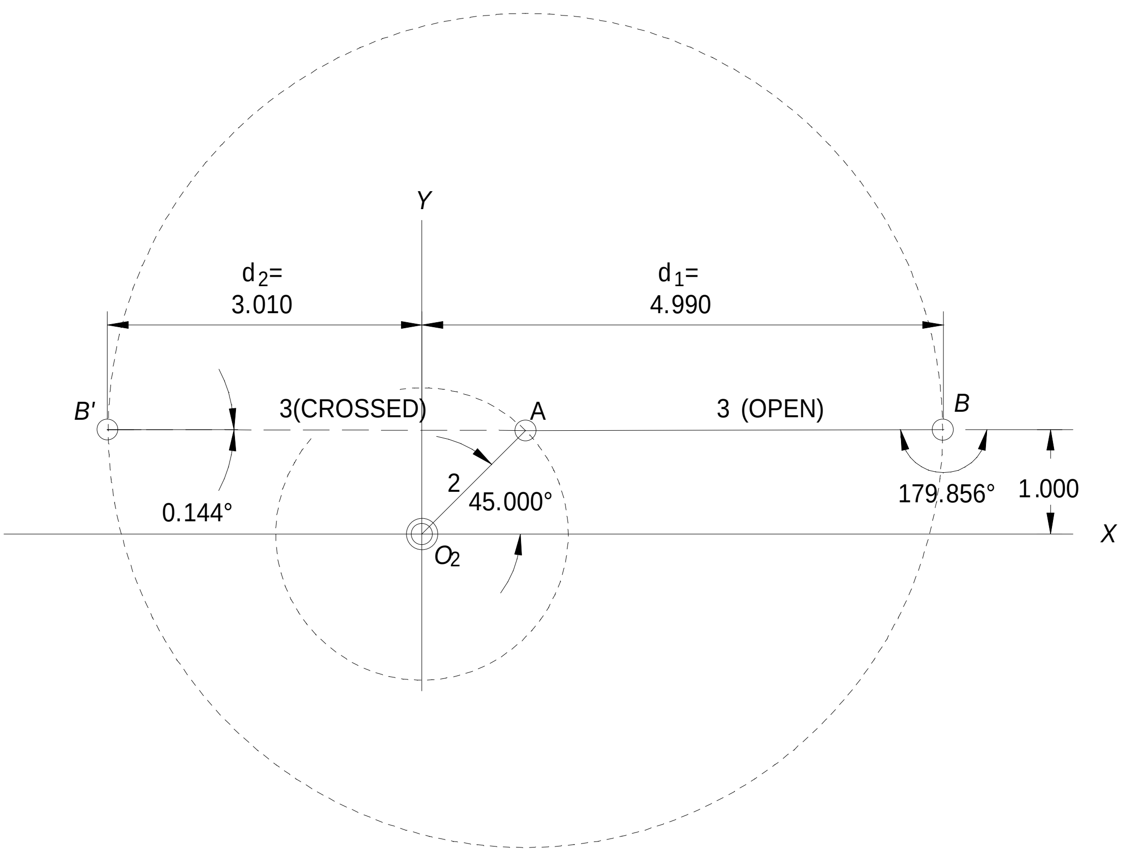
Offset $c := 1 \cdot in$ $\theta_2 := 45 \cdot deg$

Solution: See figure below for one possible solution. Also see Mathcad file P0409a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.
2. Draw link 2 to some convenient scale at its given angle.
3. Draw a circle with center at the free end of link 2 and a radius equal to the given length of link 3.
4. Draw a horizontal line through y = c (the offset).
5. The two intersections of the circle with the horizontal line (if any) are the two solutions to the position analysis problem, crossed and open. If the circle and line don't intersect, there is no solution.
6. Draw link 3 and the slider block in their two possible positions (shown as solid for open and dashed for crossed in the figure) and measure the angle θ_3 and length *d* for each circuit. From the solution below,

$\theta_{31} := 360 \cdot deg - 179.856 \cdot deg$ $\theta_{31} = 180.144\, deg$

$\theta_{32} := -0.144 \cdot deg$ $d_1 := 4.990 \cdot in$ $d_2 := -3.010 \cdot in$



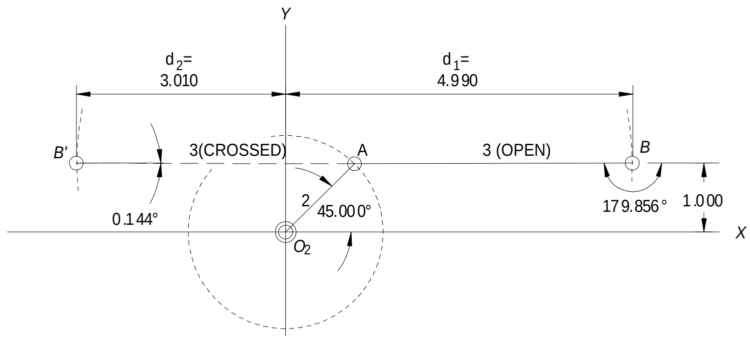
PROBLEM 4-10a

Statement: The link lengths, value of θ_2, and offset for some fourbar slider-crank linkages are defined in Table P4-2. The linkage configuration and terminology are shown in Figure P4-2. For row *a*, using the vector loop method, find all possible solutions (both open and crossed) for angles θ_3 and slider position *d*.

Given: Link 2 $a := 1.4 \cdot in$ Link 3 $b := 4 \cdot in$

Offset $c := 1 \cdot in$ $\theta_2 := 45 \cdot deg$

Solution: See Figure P4-2 and Mathcad file P0410a.



1. Determine θ_3 and *d* using equations 4.16 and 4.17.

Crossed:

$$\theta_{32} := asin\left(\frac{a \cdot sin(\theta_2) - c}{b}\right) \qquad \theta_{32} = -0.144\,deg$$

$$d_2 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{32}) \qquad d_2 = -3.010\,in$$

Open:

$$\theta_{31} := asin\left(-\frac{a \cdot sin(\theta_2) - c}{b}\right) + \pi \qquad \theta_{31} = 180.144\,deg$$

$$d_1 := a \cdot cos(\theta_2) - b \cdot cos(\theta_{31}) \qquad d_1 = 4.990\,in$$

PROBLEM 4-11a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, draw the linkage to scale and graphically find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 4 $c := 4 \cdot in$ $\gamma := 90 \cdot deg$ $\theta_2 := 30 \cdot deg$

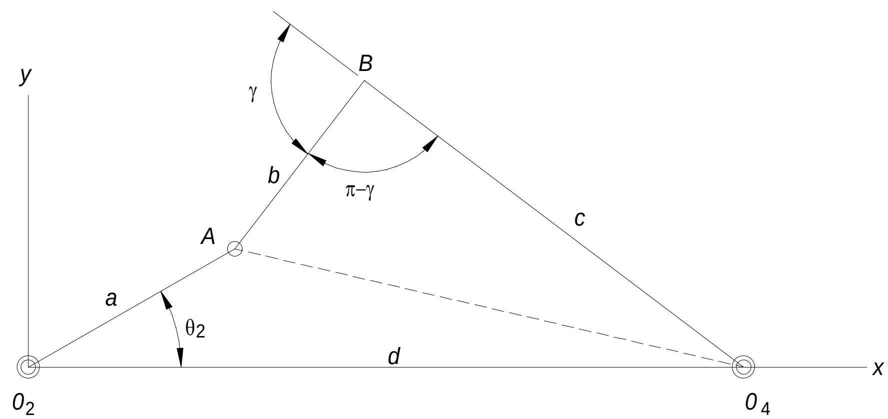
Solution: See figure below for one possible solution. Also see Mathcad file P04011a.

1. Lay out an *xy*-axis system. Its origin will be the link 2 pivot, O_2.

2. Draw link 2 to some convenient scale at its given angle.

3a. If γ = 90 deg, locate O_4 on the x-axis at a distance equal the length of link 1 (*d*) from the origin. Draw a circle with center at O_4 and radius equal to the length of link 4 (*c*). From point A, draw two lines that are tangent to the circle. The points of tangency define the location of the points *B* for the open and crossed circuits.

3b. When γ is not 90 deg there are two approaches to a graphical solution for link 3 and the location of point *B*: 1) establish the position of link 4 and the angle γ by trial and error, or 2) calculate the distance from point *A* to point *B* (the instantaneous length of link 3). Using the second approach, from triangle O_2AO_4



$$AO_4^2 = a^2 + d^2 - 2 \cdot a \cdot d \cdot cos(\theta_2)$$

and, from triangle AO_4B (for the open circuit)

$$AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos(\pi - \gamma)$$

where a, b, c, and d are the lengths of links 2, 3, 4, and 1, respectively. Eliminating AO_4 and solving for the unknown distance b for the open branch,

$$b_1 := \frac{1}{2} \cdot \left[2 \cdot c \cdot cos(\pi - \gamma) + \sqrt{(2 \cdot c \cdot cos(\pi - \gamma))^2 - 4 \cdot \left(c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot cos(\theta_2)\right)}\right]$$

$$b_1 = 1.7932\ in$$

For the closed branch: $AO_4^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot cos(\gamma)$ and

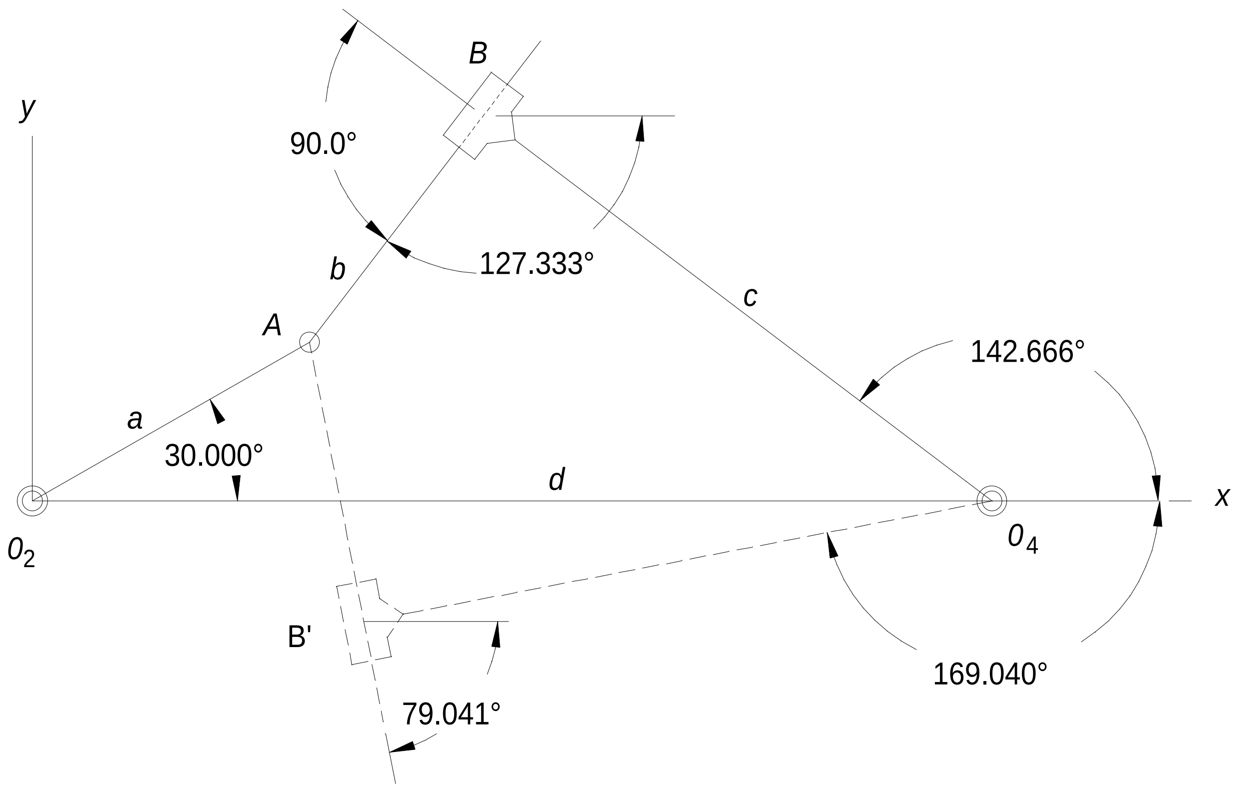
$$b_2 := \frac{1}{2} \cdot \left[2 \cdot c \cdot cos(\gamma) + \sqrt{(2 \cdot c \cdot cos(\gamma))^2 - 4 \cdot \left(c^2 - a^2 - d^2 + 2 \cdot a \cdot d \cdot cos(\theta_2)\right)}\right]$$

$b_2 = 1.7932\ in$

Draw a circle with center at point A and radius b_1. Draw a circle with center at O_4 and radius equal to the length of link 4 (c). The intersections of these two circles is the solution for the open and crossed locations of the point B.

4. Draw the complete linkage for the open and crossed circuits, including the slider. The results from the graphical solution below are:

OPEN	$\theta_{31} := -127.333 \cdot deg$	CROSSED	$\theta_{32} := 100.959 \cdot deg$
	$\theta_{41} := 142.666 \cdot deg$		$\theta_{42} := -169.040 \cdot deg$
	$R_{B1} := 3.719$ at 40.708 deg		$R_{B2} := 2.208$ at -20.146 deg



PROBLEM 4-12a

Statement: The link lengths and the value of θ_2 and γ for some inverted fourbar slider-crank linkages are defined in Table P4-3. The linkage configuration and terminology are shown in Figure P4-3. For row *a*, using the vector loop method, find both open and closed solutions for θ_3 and θ_4 and vector $\mathbf{R}_B$.

Given: Link 1 $d := 6 \cdot in$ Link 2 $a := 2 \cdot in$

Link 4 $c := 4 \cdot in$ $\gamma := 90 \cdot deg$ $\theta_2 := 30 \cdot deg$

Two argument inverse tangent

$$atan2(x, y) := \begin{cases} return\ 0.5 \cdot \pi & if\ x = 0 \wedge y > 0 \\ return\ 1.5 \cdot \pi & if\ x = 0 \wedge y < 0 \\ return\ atan\left(\left(\frac{y}{x}\right)\right) & if\ x > 0 \\ atan\left(\left(\frac{y}{x}\right)\right) + \pi & otherwise \end{cases}$$

Solution: See Mathcad file P0412a.

1. Determine the values of the constants needed for finding θ_4 from equations 4.8a and 4.10a.

$$P := a \cdot sin(\theta_2) \cdot sin(\gamma) + (a \cdot cos(\theta_2) - d) \cdot cos(\gamma) \qquad P = 1.000\ in$$

$$Q := -a \cdot sin(\theta_2) \cdot cos(\gamma) + (a \cdot cos(\theta_2) - d) \cdot sin(\gamma) \qquad Q = -4.268\ in$$

$$R := -c \cdot sin(\gamma) \qquad R = -4.000\ in \qquad T := 2 \cdot P \qquad T = 2.000\ in$$

$$S := R - Q \qquad S = 0.268\ in \qquad U := Q + R \qquad U = -8.268\ in$$

2. Use equation 4.22c to find values of θ_4 for the open and crossed circuits.

OPEN $$\theta_{41} := 2 \cdot atan2\left(2 \cdot S, -T + \sqrt{T^2 - 4 \cdot S \cdot U}\right) \qquad \theta_{41} = 142.667\ deg$$

CROSSED $$\theta_{42} := 2 \cdot atan2\left(2 \cdot S, -T - \sqrt{T^2 - 4 \cdot S \cdot U}\right) \qquad \theta_{42} = -169.041\ deg$$

3. Use equation 4.18 to find values of θ_3 for the open and crossed circuits.

OPEN $$\theta_{31} := \theta_{41} + \gamma \qquad \theta_{31} = 232.667\ deg$$

CROSSED $$\theta_{32} := \theta_{42} - \gamma \qquad \theta_{32} = -259.041\ deg$$

4. Determine the magnitude of the instantaneous "length" of link 3 from equation 4.20a.

OPEN $$b_1 := \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{41})}{sin(\theta_{41} + \gamma)} \qquad b_1 = 1.793\ in$$

CROSSED $$b_2 := \left| \frac{a \cdot sin(\theta_2) - c \cdot sin(\theta_{42})}{sin(\theta_{42} + \gamma)} \right| \qquad b_2 = 1.793\ in$$

5. Find the position vector $\mathbf{R}_B$ from the definition given on page 162 of the text.

OPEN

$$\mathbf{R_{B1}} := a\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right) - b_1\cdot\left(cos\left(\theta_{31}\right) + j\cdot sin\left(\theta_{31}\right)\right)$$

$R_{B1} := \left|\mathbf{R_{B1}}\right|$ $\qquad R_{B1} = 3.719\,in$

$\theta_{B1} := arg\left(\mathbf{R_{B1}}\right)$ $\qquad \theta_{B1} = 40.707\,deg$

CROSSED

$$\mathbf{R_{B2}} := a\cdot\left(cos\left(\theta_2\right) + j\cdot sin\left(\theta_2\right)\right) - b_2\cdot\left(cos\left(\theta_{32}\right) + j\cdot sin\left(\theta_{32}\right)\right)$$

$R_{B2} := \left|\mathbf{R_{B2}}\right|$ $\qquad R_{B2} = 2.208\,in$

$\theta_{B2} := arg\left(\mathbf{R_{B2}}\right)$ $\qquad \theta_{B2} = -20.145\,deg$